DEPENDENCE OF THE THERMAL COEFFICIENT OF ACCOMMODATION AND THE EMISSIVITY ON SURFACE ROUGHNESS

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Results are presented of a theoretical and experimental investigation of the dependence of the thermal coefficient of accommodation and emissivity on parameters characterizing surface roughness.

Multiple test determinations of the coefficients of accommodation and emissivity have shown the dependence of these parameters on the surface roughness [1, 2]. A number of attempts was recently undertaken to find the analytical expression for this dependence [3, 4].

Searches for an analytical dependence of a and ε on roughness propound primarily the question of selecting the parameter which can be used to characterize the roughness most completely from the viewpoint of its influence on the heat transmitting properties of the surface. A second, but no less important, demand on this parameter is its accessibility to measurement.

Let n be the number of collisions of gas atoms with a surface in the case of the accommodation coefficient, or the quanta of radiant energy in the case of the emissivity. The dependence of the coefficient a on n is the following [5]:

$$1-a=(1-a_0)^n. \tag{1}$$

Let us show that an analogous dependence on n will also hold for the emissivity. Indeed, ε_0 and ε can be treated as the probability of absorption of a radiant energy quantum during one or n collisions with a surface, respectively. Then $(1-\varepsilon_0)$ and $(1-\varepsilon)$ is the probability that a radiant energy quantum will not be absorbed by a surface during a single or n collisions. According to the law of multiplication of probabilities [6]

$$1-\varepsilon=(1-\varepsilon_0)^n. \tag{2}$$

Let us consider there to be scattering centers with mean height R_z in the amount of N per unit length on the surface. The mean spacing between them is $T_{mean} = 1/N$. However, in connection with the fact that the main characteristic of the height of roughness is R_a according to GOST 2789-59, and there is an approximate linear relation between R_a and R_z ($R_z = 4.5R_a$), then we shall use the parameter R_a measured by model 201 or 240 profilometers in the subsequent exposition.

It can be assumed that the number n depends on the ratios R_a/λ and T_{mean}/λ for the thermal radiation and heat transmission of gas particles by collisions. If $R_a/\lambda \geq T_{mean}/\lambda \gg 1$, then n is proportional to R_a. For $R_a/\lambda \ll 1$ $n \to 1$ is a single reflection.

The dependence of n on T_{mean}/λ is apparently more complicated. If $T_{mean}/\lambda \ll 1$, then $n \to 1$, i.e., the scattering centers are so near each other that reflection occurs from the peaks of the microroughness. If $T_{mean}/\lambda \to \infty$, then $n \to 1$ is the case of a perfectly smooth surface. When $T_{mean}/\lambda \gg 1$, but finally, then $n \sim N \sim \lambda/T_{mean}$, and this means that the dependence of n on T_{mean}/λ has a maximum.

An empirical approximation of the form

$$n=1+B\frac{R_a}{\lambda}\frac{\lambda}{T_{mean}}\exp\left(-P\frac{\lambda}{T_{mean}}\right)=1+B\frac{R_a}{T_{mean}}\exp\left(-P\frac{\lambda}{T_{mean}}\right).$$

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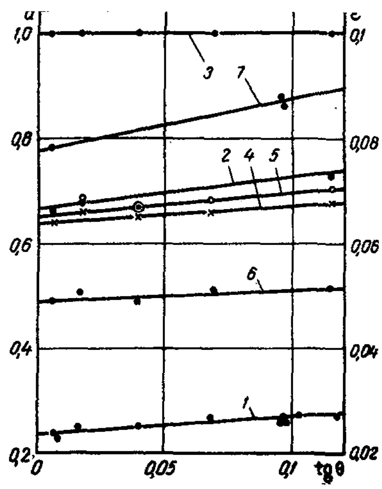

Fig. 1. Dependence of a_{eff} and ε_{eff} (curve 7) on tan θ_{mean} for different gases: 1) He; 2) Ar; 3) Xe; 4) CO_2; 5) air; 6) water vapor.

satisfies the condition listed above. The constants B and P can, in principle, be distinct for the accommodation and emissivity coefficients.

For gases λ has a magnitude on the order of 10^{-8} cm, with the exception of the temperature of liquid helium; for thermal radiation at room temperature λ is on the order of 10^{-3} cm while $T_{mean} = 1/N$ varies between $3 \cdot 10^{-2}$ and $1 \cdot 10^{-2}$ cm in the range of 8-12 classes of purity, therefore, $\lambda/T_{mean} \ll 1$ for both cases, and this means (3) will be

$$n = 1 + B\frac{R_a}{T_{mean}} = 1 + BR_aN. \tag{4}$$

The dependence obtained shows that the number of collisions n is independent of the wavelength in a broad range of temperatures ($T \geqslant 273$°K), and depends only on parameters characterizing the surface roughness.

The surface roughness profile can be considered a stationary random function, and the simplest profile characteristics can be found by the realizations of this function, by profilograms. The theory of random functions assumes a correlation method for this operation, which permits estimation of the correlation between successive points of the profile, and to obtain its frequency-dispersion characteristic.

The correlation function can be approximated by the equation [7]

$$K(\tau) = C^2R_a^2\left[\gamma \exp\left(-96\frac{\tau^2}{T_\gamma^2}\right) + \beta\cos\frac{2\pi}{T_\beta}\tau + \nu\cos\frac{2\pi}{T_\nu}\tau\right]. \tag{5}$$

The mean slope of the lateral side of a microroughness can be defined as [7]

$$\operatorname{tg}\theta = 7R_a\left(\frac{\gamma}{T_\gamma} + \frac{\beta}{T_\beta} + \frac{\nu}{T_\nu}\right), \tag{6}$$

where $\gamma/T_\gamma + \beta/T_\beta + \nu/T_\nu = 1/T_{mean}$.

Comparing (1), (4), (6) results in the expression $n = 1 + D\tan\theta$, where $D = B/7$. Therefore

$$a = 1 - (1 - a_0)^{1+D_1\operatorname{tg}\theta}$$

and

$$\varepsilon = 1 - (1 - \varepsilon_0)^{1+D_2\operatorname{tg}\theta}. \tag{7}$$

It can therefore be considered that namely $\tan\theta$ is the optimal parameter which, on the one hand, corresponds to the demands posed in the sense of taking account of the influence of surface roughness on both the accommodation coefficient and on the emissivity, and, on the other hand, can be obtained sufficiently simply from profilograms of the surface. The presence of a functional dependence on $\tan\theta$ is also noted in [3], it is true, just for the accommodation coefficient and without disclosing the form of the function.

TABLE 1. Values of the Constant D, the Accommodation and Emissivity Coefficients

Gas	a_{eff_0}	a_0	$D_{1,2}$	$\varepsilon_{\text{eff}_0}$	ε_0
He	0,23	0,375	1,35	—	—
Ar	0,66	0,8	1,36	—	—
Xe	1,0	1,0	—	—	—
Co_2	0,64	0,78	0,9	—	—
Air	0,65	0,79	0,905	—	—
Water vapor	0,48	0,66	0,60	—	—
—	—	—	1,15	0,078	0,145

The fact that the coefficients a and ε depend only on $\tan\theta$ can be shown qualitatively from several other aspects also, if the ratio of the difference between the total surface area and its geometric area ΔS to the geometric area S of the surface is taken as the roughness criterion. In turn, the quantity $\Delta S/S$ is approximately equal to the relative elongation of the profilogram because of roughness, i.e.,

$$\frac{\Delta S}{S} \approx \frac{\Delta l}{l} \approx \frac{1}{\cos\theta} - 1 \approx \frac{1}{2}\,\mathrm{tg}^2\,\theta,$$

since $\theta \ll 1$ in the range from the 6 to the 14 classes of purity.

It should be noted that the constants D_1 and D_2 in (7) depend on the law of reflection of gas atoms and radiant energy quanta from the surface at the point of incidence. Reflection of the radiant energy flux occurs according to the law that the angle of incidence equals the angle of reflection.

As regards reflection of the gas atoms, this law may not hold since the surface of a body, no matter how perfectly smooth it may be, is always "pitted" by the thermal vibrations of the solid particles and it will always be rougher for an incident atom flux than for radiant energy because the de Broglie wavelength for atoms, as has been mentioned above, is five orders less than the thermal radiation wavelength at room temperature.

It can therefore be assumed that the quantity D_1 should be greater than the quantity D_2, and the difference between these quantities can be a measure of the deviation of the law of atom reflection from the law that the angle of incidence equals the angle of reflection.

Ten pairs of coaxial tubes of 1Cr17N10T steel with 8-to 12-th class of purity were fabricated to verify the dependences (7) and the deductions following from them. The surface profile was determined on a model 201 profilometer. The correlation function $K(\tau)$, which was approximated by a dependence of the form (5), was calculated by means of the profilogram obtained, and this permitted determination of all the parameters as well as the evaluation of $\tan\theta$ in conformity with (6).

The heat flux between the coaxial tubes was determined in tests, which afforded a possibility of determining the effective emissivity (for a residual gas pressure less than 10^{-5} mm Hg) and the effective accommodation coefficient for helium, argon, xenon, air, CO_2 and water vapor:

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2} \text{ and } a_{\text{eff}} = \frac{a_1 a_2}{a_1 + a_2 - a_1 a_2}, \tag{8}$$

since for a sufficiently small gap between the tubes the problem reduces to the determination of a_{eff} and ε_{eff} for two infinite parallel planes.

Using the dependences (6) and (7), it can be shown that a_{eff} and ε_{eff} depend on $\tan\theta_1 + \tan\theta_2/2 = \tan\theta_{\text{mean}}$. For small changes in a_{eff} and ε_{eff} as a function of the roughness, and for $n \approx 1$ formula (7) can be expanded in a series in powers of $D\tan\theta$ and by limiting ourselves to the first member of the expansion we can obtain the dependence of a_{eff} and ε_{eff} on the quantity $\tan\theta_{\text{mean}}$ for the two coaxial tube surfaces

$$\begin{aligned} a_{\text{eff}} &= \frac{a_1 a_2}{a_1 + a_2 - a_1 a_2} = a_{\text{eff}_0}\left[1 - \frac{2(1-a_0)\ln(1-a_0)}{a_0(2-a_0)} D_1 \frac{(\mathrm{tg}\theta_1 + \mathrm{tg}\theta_2)}{2}\right], \\ \varepsilon_{\text{eff}} &= \frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2} = \varepsilon_{\text{eff}_0}\left[1 - \frac{2(1-\varepsilon_0)\ln(1-\varepsilon_0)}{\varepsilon_0(2-\varepsilon_0)} D_2 \frac{(\mathrm{tg}\theta_1 + \mathrm{tg}\theta_2)}{2}\right]. \end{aligned} \tag{9}$$

Graphs of the dependences of ε_{eff} and a_{eff} on $\tan\theta_{mean}$ are presented in Fig. 1. for 8-12 classes of purity. The dependence is linear. Extrapolation of the line to intersection with the ordinate axis permits determination of a_{eff} and ε_{eff_0} and evaluation of a_0 and ε_0 therefrom. Knowing a_0 and ε_0, from the slope of the lines we can determine the values of D_1 and D_2.

Values of D for the accommodation coefficient and emissivity are presented in Table 1. The quantity D_1 actually turned out to be greater than D_2 by approximately 20% but only for monatomic gases (helium and argon). For the diatomic gases (air and carbon dioxide) $D_1 < D_2$. This is possibly associated with the influence of rotational degrees of freedom of the molecules on the reflection law. This influence is particularly noticeable in the triatomic gas (water vapor) where D_1 is almost 50% less than D_2.

NOTATION

a, ε	are the thermal accommodation coefficient and emissivity, respectively (the subscripts 1, 2, eff are values for each of the heat exchange surfaces and the effective value for both surfaces together);
a_0, ε_0	are the thermal accommodation coefficient and emissivity for a single collision with the surface (n = 1);
n	is the number of collisions with the surface by the gas atoms or the radiant energy quanta;
R_z	is the mean height of the microroughness;
N	is the number of microroughnesses per unit length of surface;
T_{mean}	is the mean distance between microroughnesses;
R_a	is the arithmetic mean deviation of the microroughnesses from the mean line;
λ	is the heat radiation wavelength and the mean de Broglie wavelength for gas atoms;
B, P, D_1, D_2	are the constants in (3), (4), (7)-(9);
C	is a coefficient dependent on the nature of the microroughnesses (for ground surfaces, C = 1.25);
$K(\tau)$	is the correlation function;
τ	is the argument of the correlation function;
γ, β, ν	are the coefficients of random and periodic components of the microroughnesses (β–1 is the first order and ν-2 is the second order, where $\gamma + \beta + \nu = 1$);
T_γ	is the mean spacing between random microroughnesses;
T_β, T_ν	are the periods of the first and second order microroughnesses;
θ	is the slope of the lateral side of the microroughness (subscripts 1, 2, mean are, respectively, the values for each of the considered surfaces and their mean);
S	is the geometric surface area;
ΔS	is an increase in geometric surface area because of roughness;
l	is the length of profilogram projection on the horizontal axis;
Δl	is the difference between total profilogram length and l.

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